Looking for New Physics in B_d^0-$\overline{B_d^0}$ Mixing[1]

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Abstract. There are variety of methods which directly test for the presence of new physics in the $b \to s$ flavour-changing neutral current (FCNC), but none which cleanly probe new physics in the $b \to d$ FCNC. One possible idea is to compare the weak phase of the t-quark contribution to the $b \to d$ penguin, which is $-\beta$ in the SM, with that of B_d^0-$\overline{B_d^0}$ mixing (-2β in the SM). In this talk I show that, in fact, it is impossible to measure the weak phase of the t-quark penguin, or indeed any penguin contribution, without theoretical input. However, if one makes a single assumption involving the hadronic parameters, it is possible to obtain the weak phase. I discuss how one can apply such an assumption to the time-dependent decays $B_d^0(t) \to K^0\overline{K^0}$ and $B_s^0(t) \to \phi K_S$ in order to detect new physics in the $b \to d$ FCNC.

In the coming years, experiments at B-factories, HERA-B and hadron colliders will measure CP-violating asymmetries in B decays [1]. As always, the goal is to test the predictions of the standard model (SM). If we are lucky, there will be an inconsistency with the SM, thereby revealing the presence of new physics.

In the SM, CP violation is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. In the Wolfenstein parametrization of the CKM matrix [2], only the elements V_{ub} and V_{td} have non-negligible phases:

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{1}$$

It is convenient to parametrize V_{ub} and V_{td} as follows:

$$V_{ub} = |V_{ub}|e^{-i\gamma} \quad , \qquad V_{td} = |V_{td}|e^{-i\beta} \quad . \tag{2}$$

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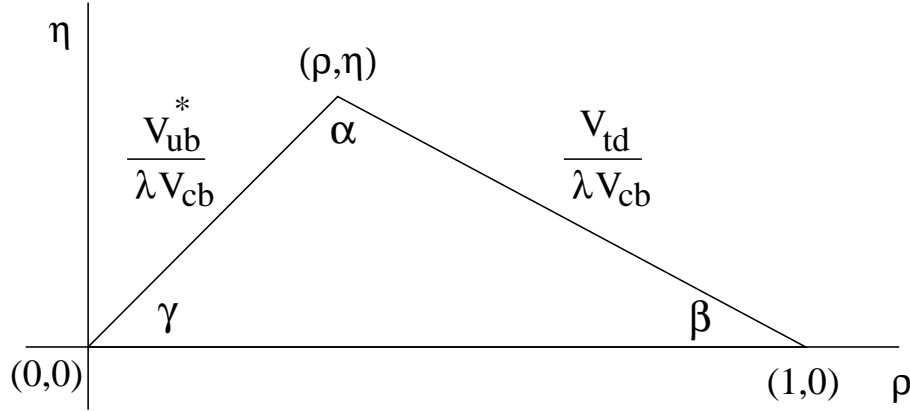
FIGURE 1. The unitarity triangle. The angles α, β and γ can be measured via CP violation in the B system.

Even though these elements are written in terms of two complex phases β and γ, it must be remembered that in fact there is only a single phase η in the CKM matrix; if η were to vanish, both β and γ would vanish as well.

The phase information in the CKM matrix can be elegantly displayed using the unitarity triangle [3]. The orthogonality of the first and third columns gives

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ , \tag{3}$$

which is a triangle relation in the complex ρ-η plane, shown in Fig. 1. The angles β and γ are two of the interior angles of the unitarity triangle, with the third angle α satisfying $\alpha + \beta + \gamma = \pi$.

There are a variety of constraints on the unitarity triangle coming from (i) the extraction of $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays, (ii) the measurements of $|V_{td}|$ and $|V_{ts}|$ in B_d^0-$\overline{B_d^0}$ and B_s^0-$\overline{B_s^0}$ mixing, and (iii) CP violation in the kaon system (ϵ). Unfortunately, there are substantial theoretical uncertainties in all of these constraints. For example, the theoretical expressions for ϵ and B_d^0-$\overline{B_d^0}$ mixing depend respectively on the bag parameter $B_K = 0.94\pm0.15$ and $f_{B_d}\sqrt{B_{B_d}} = 215\pm40$ MeV. The estimates of the magnitudes of these errors, which lie in the range 15–20%, come mainly from lattice calculations. Combining the experimental errors and theoretical uncertainties in quadrature [4], the presently-allowed region of the unitarity triangle is shown in Fig. 2. Due to the theoretical uncertainties, we do not have precise SM predictions for the CP phases α, β and γ: instead, these phases can take a range of values.

Since the hope is to find physics beyond the SM, the first question to be answered is: how can new physics affect the CP-violating asymmetries? There are two possible ways: the new physics can affect B decays or B mixing. Now, most B decays are dominated by a W-mediated tree-level diagram. In most models of new
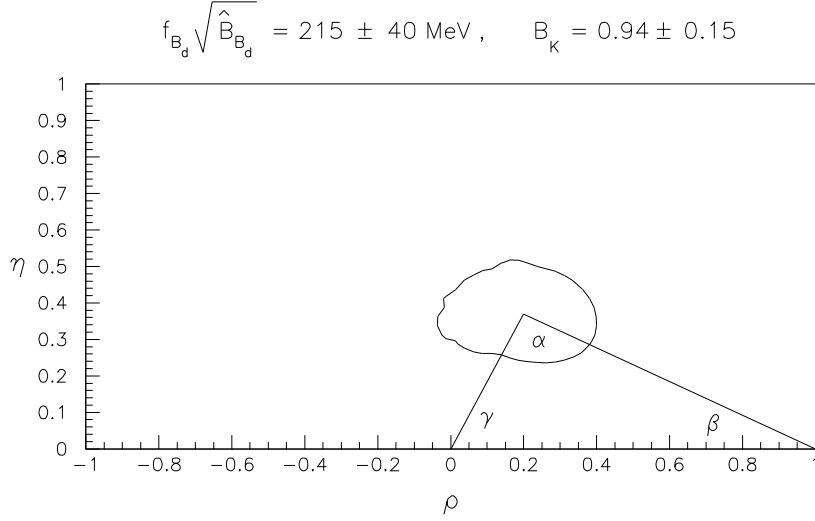
$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 215 \pm 40 \text{ MeV}, \qquad B_K = 0.94 \pm 0.15$$

FIGURE 2. Allowed region (95% C.L.) in the ρ-η plane, from a simultaneous fit to all experimental and theoretical data. The theoretical errors are treated as Gaussian for this fit. The triangle shows the best fit.

physics, there are no contributions to B decays which can compete with the SM. Thus, in general, the new physics cannot significantly affect the decays[2]. However, the CP asymmetries *can* be affected if there are new contributions to B^0-$\overline{B^0}$ mixing [6]. The presence of such new-physics contributions will affect the extraction of V_{td} and V_{ts}. And if there are new phases, the measurements of α, β and γ will also be affected. Thus, new physics enters principally through new contributions to B^0-$\overline{B^0}$ mixing [7].

Unfortunately, this creates a bit of a problem. B-factories such as BaBar and Belle will measure α, β and γ via $B_d^0(t) \to \pi^+\pi^-$ (or $\rho\pi$ [8]), $B_d^0(t) \to \Psi K_S$, and $B^\pm \to DK^\pm$ [9], respectively. Note that only the first two decays involve B^0-$\overline{B^0}$ mixing. Thus, if there is new physics, only the measurements of α and β will be affected. However, they will be affected in opposite directions [10]. That is, in the presence of a new-physics phase ϕ_{NP}, the CP angles are changed as follows: $\alpha \to \alpha + \phi_{NP}$ and $\beta \to \beta - \phi_{NP}$. The key point is that ϕ_{NP} cancels in the sum $\alpha + \beta + \gamma$, so that this sum is *insensitive* to the new physics, i.e. B-factories will always find $\alpha + \beta + \gamma = \pi$. (Note that hadron colliders do not suffer from the same problem – if γ is measured in $B_s^0(t) \to D_s^\pm K^\mp$ [11], then $\alpha + \beta + \gamma \neq \pi$ can be found if there is new physics in B_s^0-$\overline{B_s^0}$ mixing.)

Thus, B-factories cannot discover new physics via $\alpha + \beta + \gamma \neq \pi$. Still, new physics can be found if the measurements of the angles are inconsistent with the measurements of the sides. However:

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[2] There is an exception: if the decay process is dominated by a penguin diagram, rather than a tree-level diagram, then new physics *can* significantly affect the decay, see Refs. [1,5].

1. the allowed region of the unitarity triangle is still fairly large. It is conceivable that even in the presence of new physics, the triangle as constructed from the angles α, β and γ will still lie within the allowed region;

2. even if the α-β-γ triangle lies outside the allowed region, is this evidence of new physics, or have we underestimated the theoretical uncertainties which go into the constraints of the unitarity triangle (Fig. 2)?

The point is: ideally, we would like cleaner, more direct tests of the SM in order to probe for the presence of new physics.

In fact, there are such direct tests:

1. $B^{\pm} \to DK^{\pm}$ vs. $B_s^0(t) \to D_s^{\pm}K^{\mp}$: in the SM, both of these CP asymmetries measure γ. If there is a discrepancy between the value of γ as extracted from these two decays, this points to new physics in B_s^0-$\overline{B_s^0}$ mixing.

2. $B_d^0(t) \to \Psi K_S$ vs. $B_d^0(t) \to \phi K_S$: in the SM, both of these decays measure β. A discrepancy implies new physics in the $b \to s$ penguin [5].

3. $B_s^0(t) \to \Psi\phi$: in the SM, the CP asymmetry in this decay vanishes (to a good approximation). If this CP asymmetry is found to be nonzero, this again indicates the presence of new physics in B_s^0-$\overline{B_s^0}$ mixing.

There are thus several direct tests for new physics. However, note: all of these tests probe new physics in either B_s^0-$\overline{B_s^0}$ mixing or the $b \to s$ penguin, i.e. in the $b \to s$ flavour-changing neutral current (FCNC).

So this raises the question: are there any direct tests of new physics in the $b \to d$ FCNC?

Consider pure $b \to d$ penguin decays such as $B_d^0 \to K^0\overline{K^0}$ or $B_s^0 \to \phi K_S$. Such decays involve up-type quarks in the loop. If the t-quark contribution dominated, then the $b \to d$ penguin amplitude would be proportional to $V_{tb}^*V_{td}$. Recalling that the weak phase of B_d^0-$\overline{B_d^0}$ mixing is -2β and that the weak phase of V_{td} is $-\beta$, in such a case the SM would predict that (i) the CP asymmetry in $B_d^0(t) \to K^0\overline{K^0}$ vanishes, and (ii) the CP asymmetry in $B_s^0(t) \to \phi K_S$ measures $\sin 2\beta$ [12]. Any discrepancy between measurements of these CP asymmetries and their predictions would thus imply that there is new physics in either B_d^0-$\overline{B_d^0}$ mixing or the $b \to d$ penguin, i.e. in the $b \to d$ FCNC. (In the second decay, new physics in B_s^0-$\overline{B_s^0}$ mixing could also come into play, but that can be established independently, as discussed above).

However, $b \to d$ penguins are *not* dominated by the internal t-quark. The contributions of the u- and c-quarks can be as large as 20–50% of that of the t-quark [13]. In this case, the above predictions of the SM no longer hold, so that one cannot test for new physics in the $b \to d$ FCNC in this way.

So this raises a new question: are there ways of cleanly measuring the weak phase of the t-quark contribution to the $b \to d$ penguin? Unfortunately, the answer to this question is *no* [14].

To see this, consider the general form of the amplitude for the $b \to d$ penguin. There are three terms, corresponding to the contributions of the three internal up-type quarks:

$$P = P_u \, V_{ub}^* V_{ud} + P_c \, V_{cb}^* V_{cd} + P_t \, V_{tb}^* V_{td} \; , \tag{4}$$

and recall that $V_{ub} \sim e^{-i\gamma}$ and $V_{td} \sim e^{-i\beta}$.

Using the unitarity relation of Eq. 3, the u-quark piece can be eliminated in the above equation, allowing us to write

$$P = \mathcal{P}_{cu} \, e^{i\delta_{cu}} + \mathcal{P}_{tu} \, e^{i\delta_{tu}} e^{-i\beta} \; , \tag{5}$$

where δ_{cu} and δ_{tu} are strong phases. Now imagine that there were a method in which a series of measurements allowed us to cleanly extract β using the above expression. In this case, we would be able to express $-\beta$ as a function of the observables.

On the other hand, we can instead use the unitarity relation to eliminate the t-quark contribution in Eq. 4, yielding

$$P = \mathcal{P}_{ct} \, e^{i\delta_{ct}} + \mathcal{P}_{ut} \, e^{i\delta_{ut}} e^{i\gamma} \; . \tag{6}$$

Comparing Eqs. 5 and 6, we see that they have the same form. Thus, the same method which allowed us to extract $-\beta$ from Eq. 5 should be applicable to Eq. 6, allowing us to obtain γ. That is, we would be able to write γ as *the same function* of the observables as was used for $-\beta$ above! But this implies that $-\beta = \gamma$, which clearly doesn't hold in general.

Due to the ambiguity in the parametrization of the $b \to d$ penguin — which I will refer to as the *CKM ambiguity* — we therefore conclude that one cannot cleanly extract the weak phase of any penguin contribution. Indeed, it is *impossible* to cleanly test for the presence of new physics in the $b \to d$ FCNC.

Nevertheless, it is interesting to examine some candidate methods and see how they fail. For example, consider the time-dependent rate for the decay $B_d^0(t) \to K^0\overline{K^0}$. This can be written

$$\Gamma(B_d^0(t) \to K^0\overline{K^0}) = e^{-\Gamma t} \left[\frac{|A|^2 + |\bar{A}|^2}{2} + \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta M t) \right.$$
$$\left. - \mathrm{Im}\left(e^{-2i\beta} A^* \bar{A} \right) \sin(\Delta M t) \right], \tag{7}$$

where $A \equiv A(B_d^0 \to K^0\overline{K^0})$ and $\bar{A} \equiv A(\overline{B_d^0} \to K^0\overline{K^0})$. The measurement of this time-dependent decay rate allows one to extract the magnitudes and relative phase of $e^{i\beta}A$ and $e^{-i\beta}\bar{A}$. Using the form of the $b \to d$ penguin given in Eq. 5, we have

$$e^{i\beta} A = e^{i\beta} \left[\mathcal{P}_{cu} \, e^{i\delta_{cu}} + \mathcal{P}_{tu} \, e^{i\delta_{tu}} e^{-i\beta'} \right], \tag{8}$$

where in the last term I have written the weak phase as β' to allow for the possibility of new physics in the $b \to d$ FCNC. There are thus 5 measurable parameters: $\mathcal{P}_{cu}$,

$\mathcal{P}_{tu}$, $\delta_{cu} - \delta_{tu}$, β, and $\theta_{NP} \equiv \beta' - \beta$. However, there are only 4 measurements: the coefficients of the 3 time-dependent functions $[1, \cos(\Delta M t), \sin(\Delta M t)]$ in Eq. 7, and one independent measurement of β. Therefore, as argued above, there are not enough measurements to determine all the theoretical parameters. More to the point, there is one more theoretical unknown than there are measurements.

In fact, one can examine a variety of other techniques: $B \to \pi\pi$ isospin analysis [15], Dalitz plot analysis of $B \to 3\pi$ [8], angular analysis of the decay of a neutral B-meson to two vector mesons [16], and a combined isospin + angular analysis of $B \to \rho\rho$. In all cases there is one more unknown than there are measurements. From this we can therefore conclude the following: due to the CKM ambiguity, if we wish to test for the presence of new physics in the $b \to d$ FCNC by comparing the weak phase of B_d^0-$\overline{B_d^0}$ mixing with that of the t-quark contribution to the $b \to d$ penguin, it is necessary to make a single assumption about the theoretical (hadronic) parameters describing the decay.

As an example of such an assumption, consider again the two decays $B_d^0(t) \to K^0\overline{K^0}$ and $B_s^0(t) \to \phi K_s$. Recall that we can write the $B_d^0 \to K^0\overline{K^0}$ amplitude as

$$e^{i\beta} A_d^{K^0\overline{K^0}} = \mathcal{P}_{cu} \, e^{i\delta_{cu}} e^{i\beta} + \mathcal{P}_{tu} \, e^{i\delta_{tu}} e^{-i(\beta'-\beta)} \ . \tag{9}$$

Assuming that there is no new physics in B_s^0-$\overline{B_s^0}$ mixing, we can write the $B_s^0 \to \phi K_s$ amplitude as

$$A_s^{\phi K_S} = \tilde{\mathcal{P}}_{cu} \, e^{i\tilde{\delta}_{cu}} + \tilde{\mathcal{P}}_{tu} \, e^{i\tilde{\delta}_{tu}} e^{-i\beta'} \ . \tag{10}$$

The tildes are added to distinguish the parameters in the decay $B_s^0 \to \phi K_S$ from those in $B_d^0 \to K^0\overline{K^0}$. There are two reasons. First, in the B_s^0 decay, we have a spectator s-quark instead of a d-quark. And second, there are colour-allowed electroweak penguin contributions to $B_s^0 \to \phi K_S$ while there are none in $B_d^0 \to K^0\overline{K^0}$.

From the above, we see that there are 8 theoretical parameters describing these two decays: $\mathcal{P}_{cu}$, $\mathcal{P}_{tu}$, $\tilde{\mathcal{P}}_{cu}$, $\tilde{\mathcal{P}}_{tu}$, β, β', $\delta_{cu} - \delta_{tu}$, and $\tilde{\delta}_{cu} - \tilde{\delta}_{tu}$. However there are only 7 experimental measurements: the magnitudes and relative phase of $e^{i\beta} A_d^{K^0\overline{K^0}}$ and $e^{-i\beta} \bar{A}_d^{K^0\overline{K^0}}$, the magnitudes and relative phase of $A_s^{\phi K_S}$ and $\bar{A}_s^{\phi K_S}$, and an independent measurement of β. If we wish to determine the theoretical parameters, we therefore need to make an assumption.

In Ref. [17], the following assumption is made. Defining $r \equiv \mathcal{P}_{cu}/\mathcal{P}_{tu}$ and $\tilde{r} \equiv \tilde{\mathcal{P}}_{cu}/\tilde{\mathcal{P}}_{tu}$, it is assumed that $r = \tilde{r}$. How good is this assumption? Writing

$$r = \left| \frac{P_c - P_u}{P_t - P_u + P_{EW}^C} \right| \ , \qquad \tilde{r} = \left| \frac{\tilde{P}_c - \tilde{P}_u}{\tilde{P}_t - \tilde{P}_u + \tilde{P}_{EW} + \tilde{P}_{EW}^C} \right| \ , \tag{11}$$

we note the following. Since the spectator-quark effects cancel in the ratio in $\tilde{r}$, the principle difference between r and $\tilde{r}$ is due to the presence of the colour-allowed electroweak penguin contribution in the denominator of $\tilde{r}$. Since $\tilde{P}_{EW}/\tilde{P}_t \simeq 20\%$,

we therefore conclude that r and $\tilde{r}$ are equal to within roughly 20%. Taking $r = \tilde{r}$ is therefore a reasonable assumption.

With this assumption, we now have an equal number of theoretical unknowns and experimental measurements, and can therefore solve for β and β' independently. In this way we can test for the presence of new physics in the $b \to d$ FCNC. Note also that the assumption of $r = \tilde{r}$ holds only within a particular parametrization of the $b \to d$ penguin, so that the CKM ambiguity is lifted.

There are, in fact, other methods where an assumption can be used to measure the weak phase of the t-quark contribution to the $b \to d$ penguin. My collaborators and I are currently examining such methods.

To summarize: if the unitarity triangle as constructed from measurements of the CP angles α, β and γ disagrees with that constructed from measurements of the sides, we may deduce that there is new physics in B_d^0-$\overline{B_d^0}$ mixing. However, it may be that the discrepancy is due not to the presence of new physics, but rather to an underestimate of the theoretical uncertainties which enter into the constraints on the unitarity triangle. For this reason, it is preferable to have direct tests for new physics.

There are, in fact, several such direct tests, but they all probe new physics in the $b \to s$ FCNC. One possibility of searching for new physics in the $b \to d$ FCNC is the following: in the SM the weak phase of B_d^0-$\overline{B_d^0}$ mixing is -2β, while that of the t-quark contribution to the $b \to d$ penguin is $-\beta$. A comparison of these two weak phases might reveal new physics in the $b \to d$ FCNC.

Unfortunately, due to the ambiguity in parametrizing the $b \to d$ penguin, it is impossible to cleanly measure the weak phase of the t-quark contribution to the $b \to d$ penguin. In order to measure this phase, it is necessary to make an assumption about the hadronic parameters. I presented one example involving the two decays $B_d^0(t) \to K^0\overline{K^0}$ and $B_s^0(t) \to \phi K_s$, but there are other methods. With such an assumption it is possible to detect the presence of new physics in the $b \to d$ FCNC.

Acknowledgments

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